Review for Students Entering

Calculus

Dear Student,

You are receiving this summer packet as a review of previously covered math topics needed to be successful in the upcoming math class you will be taking in the 2018-19 school year. The SCVTS Math Department requires that students complete this packet and bring it, with work shown, to school on the first day. Students are requested to use pencil, and show their work in the packet or on lined paper to accompany the packet. The packet will be reviewed, and there will be a test on the material in the packet on the fourth day of the semester. The Math Department recommends that students in Algebra 1 and Geometry have the TI-30XS scientific calculator, and students in Algebra 2 and above have a TI-83 or TI84 graphing calculator.

In addition to the examples shown in the packet, you are encouraged to use the many resources available at the following websites:

https://www.khanacademy.org

http://www.purplemath.com

http://www.mathisfun.com

https://www.desmos.com   is a free online graphing calculator also available as a free mobile app for most smart phones.

http://www.youtube.com/user/profrobbob   is a YouTube channel featuring video tutorials for a variety of high school level mathematics

Using the search engine on YouTube will also result in plenty of video tutorials that may be useful as well.

Students may turn in the packet early by dropping it off in the main office at CTHS.

Any questions may be directed via email to any of the following teachers in the math department. For incoming freshman please contact Nicole Kopp or Eric Lockwood.

Nicole Kopp	nkopp@scvts.org
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<u>Grading Criteria:</u>

The completion of the packet will be counted as **two homework grades**. If it is not turned in by the first day of school, there will be a 10 point late penalty per day, and will not be accepted after the first week of the semester. The packet will be graded based on the percentage completed. To avoid earning a 0, students should show all their work, and complete at least half of the math packet. As a reminder, homework is counted as 20%, and tests are worth 40% of the marking period grade.

Parent/ Guardian Acknowledgement Statement

I understand that the purpose of the summer packet is for my child to review the topics they have already mastered in previous math classes and therefore will be prepared to take the class they are currently enrolled in.

____

(Parent/Guardian Signature)
Date

Formula Sheet

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

Logarithms:

$y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Fractional exponent: $\sqrt[b]{x^e} = x^{\frac{e}{b}}$

Negative Exponents: $x^{-n} = 1/x^n$

The Zero Exponent: $x^0=1$, for x not equal to 0.

Multiplying Powers

Multiplying Two Powers of the Same Base:

$(x^a)(x^b) = x^{(a+b)}$

Multiplying Powers of Different Bases:

$(xy)^a = (x^a)(y^a)$

Dividing Powers

Dividing Two Powers of the Same Base:

$(x^a)/(x^b) = x^{(a-b)}$

Dividing Powers of Different Bases:

$(x/y)^a = (x^a)/(y^a)$

Slope-intercept form: $y = mx + b$

Point-slope form: $y = m(x - x_1) + y_1$

Standard form: $Ax + By + C = 0$

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Function

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

6. $f(2) =$ ____________
7. $g(-3) =$ ____________
8. $f(t+1) =$ __________
9. $f[g(-2)] =$ __________
10. $g[f(m+2)] =$ ___________
11. $\dfrac{f(x+h) - f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ ___________
13. $f\left(\dfrac{2\pi}{3}\right) =$ ______________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

14. $h[f(-2)] =$ _______
15. $f[g(x-1)] =$ _______
16. $g[h(x^3)] =$ _______

Find $\frac{f(x+h)-f(x)}{h}$ for the given function *f*.

17. $f(x)=9x+3$

18. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$x-\text{int}.\ (Let\ y=0)$	$y-\text{int}.\ (Let\ x=0)$
$0=x^2-2x-3$	$y=0^2-2(0)-3$
$0=(x-3)(x+1)$	$y=-3$
$x=-1\ or\ x=3$	$y-\text{intercept}\ (0,-3)$
$x-\text{intercepts}\ (-1,0)\ and\ (3,0)$	

Find the x and y intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Systems

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *and* $x = 5$

Plug $x = 3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection $(5,4), (5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ *(The rest is the same as previous example)*

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *or* $x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23. $x + y = 8$
$4x - y = 7$

24. $x^2 + y = 6$
$x + y = 4$

Interval Notation

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1,7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1\ge 0$

28. $-4\le 2x-3<4$

29. $\frac{x}{2}-\frac{x}{3}>5$

<u>**Domain and Range**</u>

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

<u>**Inverses**</u>

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that:
$f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ *and* $g(x) = 4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$g(f(x)) = 4\left(\frac{x-9}{4}\right)+9$$
$$= x-9+9$$
$$= x$$

$$f(g(x)) = \frac{(4x+9)-9}{4}$$
$$= \frac{4x+9-9}{4}$$
$$= \frac{4x}{4}$$
$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

36. $f(x) = \frac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

37. $f(x) = 9 - x^2, x \geq 0$ $\quad g(x) = \sqrt{9-x}$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.	Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

46. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

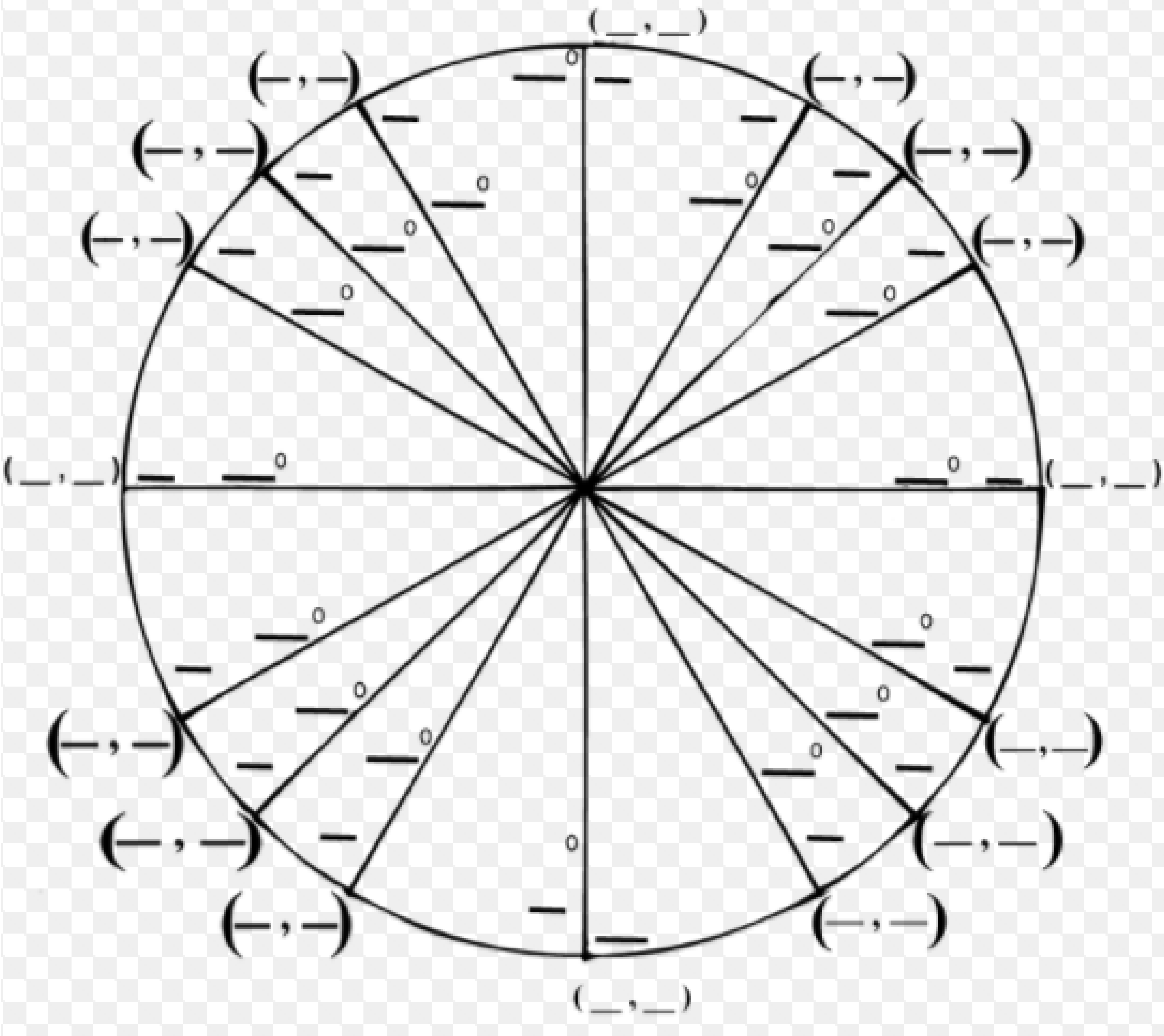
47. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

48. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians

Unit Circle, Fill in the blank



Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

71. $f(x) = \frac{1}{x^2}$

72. $f(x) = \frac{x^2}{x^2 - 4}$

73. $f(x) = \frac{2 + x}{x^2(1 - x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

74. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

75. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

76. $f(x) = \frac{4x^5}{x^2 - 7}$

Laws of Exponents

Write each of the following expressions in the form $ca^p b^q$ where c, p and q are constants (numbers).

75. $\frac{(2a^2)^3}{b}$

76. $\sqrt{9ab^3}$

(Hint: $\sqrt{x} = x^{1/2}$)

77. $\frac{a(2/b)}{3/a}$

78. $\frac{ab-a}{b^2-b}$

79. $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$

80. $\left(\frac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^2 \left(\frac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)$

Laws of Logarithms

Simplify each of the following:

81. $\log_2 5 + \log_2(x^2-1) - \log_2(x-1)$

82. $2\log_2 9 - \log_2 3$

83. $3^{2\log_3 5}$

84. $\log_{10}(10^{\frac{1}{2}})$

85. $\log_{10}(\frac{1}{10^x})$

86. $2\log_{10}\sqrt{x} + \log_{10} x^{\frac{1}{3}}$

Solving Exponential and Logarithmic Equations

Solve for x. (DO NOT USE A CALCULATOR)

87. $5^{(x+1)} = 25$

88. $\frac{1}{3} = 3^{2x+2}$

89. $\log_2 x^2 = 3$

90. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

Factor Completely

91. $x^6 - 16x^4$

92. $4x^3 - 8x^2 - 25x + 50$

93. $8x^3 + 27$

94. $x^4 - 1$

Solve the following equations for the indicated variables:

95. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1,\ for\ a.$

96. $V = 2(ab + bc + ca),\ for\ a.$

97. $A = 2\pi r^2 + 2\pi rh,\ for\ positive\ r.$

Hint: use quadratic formula

98. $A = P + xrP,\ for\ P$

99. $2x - 2yd = y + xd,\ for\ d$

100. $\frac{2x}{4\pi} + \frac{1-x}{2} = 0,\ for\ x$

Solve the equations for x:

101. $4x^2 + 12x + 3 = 0$

102. $2x + 1 = \frac{5}{x+2}$

103. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

Polynomial Division

104. $(x^5 - 4x^4 + x^3 - 7x + 1) \div (x + 2)$

105. $(x^5 - x^4 + x^3 + 2x^2 - x + 4) \div (x^3 + 1)$