Pre-Calculus Summer Packet Instructions

Dear Student,

You are receiving this summer packet as a review of previously covered math topics needed to be successful in the upcoming math class you will be taking the 2018-19 school year. The SCVTS Math Department requires that students complete this packet and bring it, with work shown, to school on the first day. Students are requested to use pencil, and show their work in the packet or on lined paper to accompany the packet. The packet will be reviewed, and there will be a test on the material in the packet on Friday, September 7th. The Math Department recommends that students in Algebra 1 and Geometry have the TI-30XS scientific calculator, and students in Algebra 2 and above have a TI-83 or TI-84 graphing calculator.

In addition to the examples shown in the packet, you are encouraged to use the many resources available at the following websites:

https://www.khanacademy.org

http://www.purplemath.com

http://www.mathisfun.com

https://www.desmos.com is a free online graphing calculator also available as a free mobile app for most smart phones.

http://www.youtube.com/user/profrobbob is a YouTube channel featuring video tutorials for a variety of high school level mathematics

Using the search engine on YouTube will also result in plenty of video tutorials that may be useful as well.

Students may turn in the packet early by dropping it off in the main office at CTHS.

Any questions may be directed via email to any of the following teachers in the math department. For incoming freshman please contact Nicole Kopp or Eric Lockwood.

Nicole Kopp	nkopp@scvts.org
Jessica Crim	jcrim@scvts.org
Eric Lockwood	elockwood@scvts.org
Jen Roberts	jroberts@scvts.org
Eric Walter	ewalter@scvts.org

Grading Criteria:

The completion of the packet will be counted as **two homework grades**. If it is not turned in by the first day of school, there will be a 10 point late penalty per day, and will not be accepted after Monday, September 10th. The packet will be graded based on the percentage completed. To avoid earning a 0, students should show all their work, and complete at least half of the math packet. As a reminder, homework is counted as 20%, and tests are worth 40% of the marking period grade.

Parent/Guardian Acknowledgment

Dear Parent/Guardian,

Please read and sign the following.

I understand that the purpose of the summer packet is for my child to review the topics they have already mastered in previous math classes and therefore will be prepared to take the class they are currently enrolled in.

_____

(Parent/Guardian Signature) Date

Write your final answer on the line for the corresponding problem. Show all work.

1) List $5, -1\frac{1}{5}, 3\frac{4}{5}, -\frac{3}{5}, \text{and } 2\frac{2}{5}$ from least to greatest. 1) _____

2) Simplify $[2^3 + 4(7 - 3)] \div 8$ 2) _____

3) Evaluate $\frac{a^2+b^2}{a-b} + \frac{bc}{a}$ if a = 5, b = 3, and c = 15 3) _____

4) Simplify $\frac{-(7-9)(7+9)}{(2-6)\cdot 4^2}$ 4) _____

5) Simplify $\frac{16a^3-4a^2+64a+36}{4}$ 5) _____

<u>Equations</u>
Solve for x.

6) $\frac{3}{5}x + 3 = 2x - 11$ 7) $2(x - 1) = 5 - (3 - 2x)$ 6) _____

7) _____

8) $2(2x - 9) = 7x - 60$ 9) $\frac{6x-2(x-4)}{3} = 8$ 8) _____

9) _____

Word Problems

For these problems (and for ALL word problems) be sure to begin by reading carefully. Then, identify what the variable will stand for and be very specific. For example, state the variable will represent "cost of adult tickets" or "number of tickets sold" instead of only stating "tickets." The next step is to write an equation and solve it. Finally, answer the questions asked in the problem.

10) A theater has 600 tickets to sell for a show. Of these tickets, 225 sell for $2 a piece more than the others. If all tickets are sold and $2250 is taken for the show, what price is each type of ticket?

10) _____

11) A music store offers piano lessons at a discount for customers buying new pianos. The costs for lessons and a one-time fee for materials (including music books, CDs, software, etc.) are 6 lessons for $300, and 12 Lessons for $480. What is the cost of each lesson and the one-time fee for materials?

11) _____

Inequalities

Solve each inequality. Remember that when you multiply or divide both sides of an inequality by a negative number, the direction of the inequality symbol changes.

12) $5x - 4 \geq 3x$

13) $3(5x + 4) < 13x - 10$

12) _____

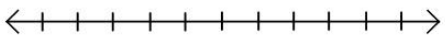
13) _____

Solve the compound inequalities (conjunctions and disjunctions). Plot your solution on a number line to help you interpret that solution correctly.

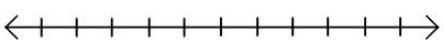
14) $-3 < 3 + 2w < 6$

15) $2 < \frac{x}{5}$ or $2x + 4 \leq -6$

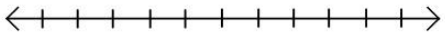
14) _____



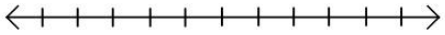
15) _____



16) $-3 \leq -2(x-3) < 6$ 17) $2x+3>1$ or $5x-9 \leq 6$ 16) _____



17) _____



<u>Absolute Value Equations and Inequalities</u>

Absolute value problems will always be broken down into two separate equations (or inequalities) *AFTER* the absolute value expression is isolated. Here is an example of each:

<u>Absolute Value Equation:</u>

$$3-|2x-3|=1$$
$$-3 \quad -3$$
$$-|2x-3|=-2$$
$$\div -1 \quad \div -1$$
$$|2x-3|=2$$

Split in two equations:

$2x-3=2$ $\quad$ $2x-3=-2$

$+3 \ +3$ $\quad$ $+3 \ +3$

$2x=5$ $\quad$ $2x=1$

$\div 2 \ \div 2$ $\quad$ $\div 2 \ \div 2$

$x = {}^{5}/_{2}$ or $x = {}^{1}/_{2}$

<u>Absolute Value Inequality:</u>

$$|3k-2|<4$$

Split in two inequalities:

$3k-2<4$ and $3k-2>-4$

$+2 \ +2$ $\quad$ $+2 \ +2$

$3k<6$ $\quad$ $3k>-2$

$\div 3 \ \div 3$ $\quad$ $\div 3 \ \div 3$

$k<2$ $\quad$ $k>-{}^{1}/_{3}$

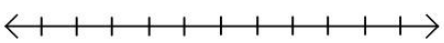
Solve each equation. Your final answer should contain <u>two solutions</u>.

18) $|4x-10|=12$ 19) $4|x-7|=2$ 18) _____

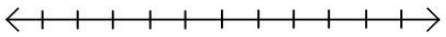
19) _____

Solve and <u>graph</u> on a number line the solution set of each open sentence. Reminder: When setting up the absolute value inequality as two separate inequalities, a less than inequality (<, <u>≤</u>**) becomes a conjunction** (use *AND*) **and a greater than inequality** (>, <u>≥</u>**) becomes a disjunction** (use *OR*).

20) $|2x+5| \leq 7$ 20) _____



21) $|w - 5| > 3$ 21) _____



Linear Equations

All coordinate points are ordered as (*x*, *y*).

The formula for the slope of the line between two points (x_1, y_1) and (x_2, y_2) is: $m = \frac{y_2 - y_1}{x_2 - x_1}$

There are three forms of the equation of a line:

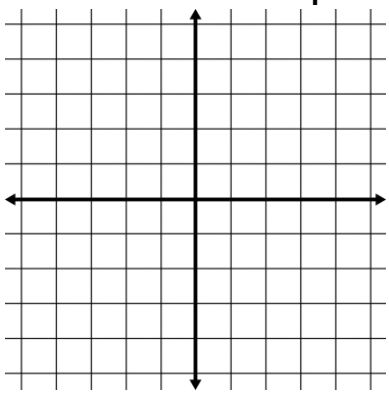
Slope-Intercept	**y = mx + b**	**m = slope, b = y-intercept**
Standard Form:	**Ax + By = C**	**A, B, and C are integers**
Point-Slope Form:	**$y - y_1 = m(x - x_1)$**	**m = slope, (x_1, y_1) is a point on the line**

22) Determine the constant *k* so that (-2, 5) will be a solution of $2x + ky = 3(k - 1)$.

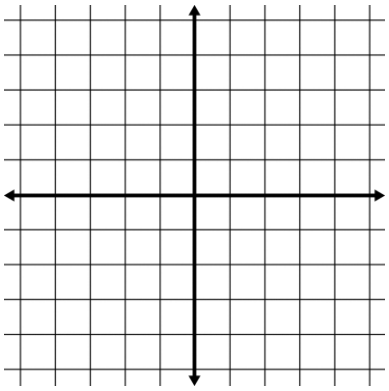
22) _____

Graph each equation on a coordinate plane.

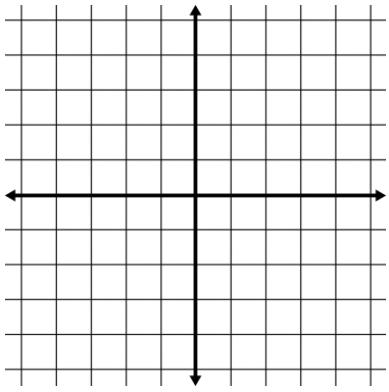
23) $y - 3 = 0$



24) $3x - y = -5$



25) $x + y = 0$



26) Find the slope of the line that passes through (-2, -7) and (0, 9) 26) _____

27) Find the slope of the line $3x + 4y = 9$ 27) _____

28) Find the equation of the line that passes through (2, 5) and has slope = -2

28) _____

29) Find the equation of the line that passes through $(-3, 0)$ and (0, 6).

29) _____

Reminder: Parallel lines have the SAME slope, and perpendicular lines have OPPOSITE RECIPROCAL slopes.
Write the equation of the line according to the directions in each problem.

30) Passes through (4, -3) and parallel to $3x - y = -5$ 30) _____

31) Passes through (4, -3) and perpendicular to $3x - y = -5$ 31) _____

Systems of Equations

Solve the systems of equations using either substitution or elimination method. (You may check your solution using www.desmos.com or by plugging in your solution to both of the original equations.)

32) $\begin{cases} 3x + 4y = 2 \\ -5x + 4y = -2 \end{cases}$

33) $\begin{cases} 2x + 3y = 4 \\ 5x + 4y = 3 \end{cases}$

32) _____

33) _____

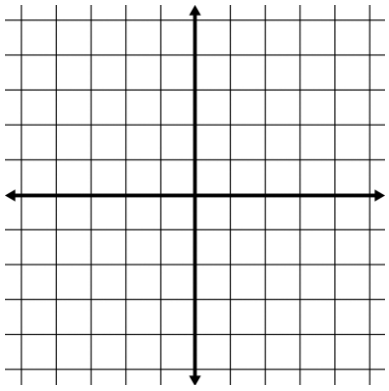
Linear Inequalities & Systems of Inequalities

Follow the steps to graph linear inequalities correctly:

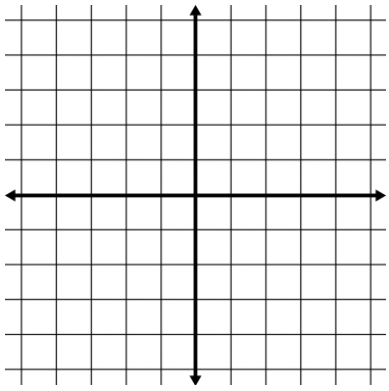
1. Solve each inequality for *y* to transform the equation into slope-intercept form, and graph the line based upon the slope and y-intercept. (When dividing by a negative, remember to flip the inequality symbol!)
2. The boundary line will be **DASHED** for < or >, and **SOLID** for $\leq$ or $\geq$.
3. Shade the area **ABOVE** the line for $y >$ or $y \geq$, and **BELOW** the line for $y <$ or $y \leq$

Graph the linear inequalities.

34) $x + 2y \leq 2$

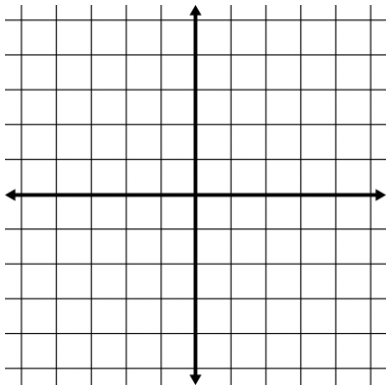


35) $2x - 3y < 6$



Graph the system of inequalities. Identify the solution region by designating it with an "S."

36) $\begin{cases} x - 3y > 6 \\ x + y \leq 2 \end{cases}$



Simplifying Expressions

Rules to Remember:

Adding & Subtracting Terms must have the same variable(s) & exponents!

Examples**:** $3x^2 + 5x^2 = 8x^2$

$2xy^2 + 3x^2y$ are NOT like terms, so cannot be added

Multiplying variables, add exponents: $a^m \cdot a^n = a^{m+n}$

To multiply binomials use FOIL. Example: $(5x - 3)(2x + 1)$

Multiply the FIRST terms, OUTER terms, INSIDE terms, and LAST terms

$$(5x)(2x) + (5x)(1) + (-3)(2x) + (-3)(1)$$
$$= 10x^2 + 5x - 6x - 3$$
$$= 10x^2 - x - 3$$
$$(5x - 3)(2x + 1) = 10x^2 - x - 3$$

37) $4(x^2 + 3) + 5(2 - 3x^2)$

38) $(-6x^2r^2y)(5xr^3y^2)$

37) _____

38) _____

39) $(2x - y)(3x + 2y)$

40) $b^2x^2(2b + x^2)$

39) _____

40) _____

Find the Greatest Common Factor (GCF) and Least Common Multiple (LCM) of the following.

41) $420, 504$

42) $15n^2m^3,\ 25nm^2,\ 45n^2m$

41) _____

42) _____

Factoring Polynomials

Reminders: Always look for a GCF to factor out first!

Forms to Factor:

1. **Difference of Squares $a^2 - b^2 = (a + b)(a - b)$**
   Example**:** $4x^2 - 81 = (2x + 9)(2x - 9)$
2. **Trinomial $x^2 + bx + c = (x + m)(x + n)$ - Find factors *m* & *n* of 'C' that add up to 'B'**
   Example**:** $x^2 - x - 42 = (x - 7)(x + 6)$
3. **Trinomial $ax^2 + bx + c$ – Find factors of 'A*C' that add up to 'B'. Rewrite binomials as $(ax + m)(ax + n)$ then divide out any common factor of each binomial.**
   Example**:** $10x^2 + 11x - 8 = (10x + 16)(10x - 5)$ Divide $(10x + 16)$ by 2, and $(10x - 5)$ by 5
   $= (5x + 8)(2x - 1)$
4. **Four Terms – Grouping: Group the first two terms together and the last two terms together. Factor out the GCF for each binomial. The GCF from each binomial will become one factor and remaining binomial will be the other factor.**
   Example**:** $x^3 + 7x^2 + 2x + 14 = (x^3 + 7x^2) + (2x + 14)$
   $= x^2(x + 7) + 2(x + 7)$
   $= (x^2 + 2)(x + 7)$ is the final factored form.

Factor each polynomial completely.

43) $64r^2 + 16r + 1$

44) $6d^2 + 3de + 10d + 5e$

43) _____

44) _____

45) $a^2 + ab - 6b^2$

46) $9x^2 - 15x + 6$

45) _____

46) _____

47) $49x^2 - 16$

48) $x^2 - 16x + 64$

47) _____

48) _____

Quadratic Equations

There are 3 techniques for solving quadratic equations:

1. **Factoring to use Zero Product Property**
   Once the quadratic polynomial is factored and it's equal to zero, set each factor equal to zero to solve for the variable.
2. **Completing the Square**
3. **Using the Quadratic Formula**
   For any quadratic equation in the form: $ax^2 + bx + c = 0$ the solutions can be found using the formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve each equation.

49) $(x + 5)(2x - 1) = 0$

50) $x^2 - 12x - 10 = 0$

49) _____

50) _____

51) $6x^2 - x - 2 = 0$

51) _____

Rational Expressions

Simplifying rational expressions requires factoring the numerator and denominator, and then you may cancel out any like factors.

Example: $\frac{3h-9}{h^2-8h+15} = \frac{3(h-3)}{(h-3)(h-5)} = \frac{3}{h-5}$

Multiplying rational expressions requires you to multiply straight across, make sure to reduce your final answer.

Example: $\frac{2x^2y}{3x^2y} \cdot \frac{3xy}{4y} = \frac{6x^3y^2}{12x^2y^2} = \frac{x}{2}$

Dividing rational expressions requires to "Keep, Change, Flip." Keep the first fraction the same, change the division symbol to multiplication, and then flip the second fraction.

Example: $\frac{6a^3}{4b^2} \div \frac{2a^2}{12b^2} = \frac{6a^3}{4b^2} \cdot \frac{12b^2}{2a^2} = \frac{72a^3b^2}{8a^2b^2} = 9a$

Adding and subtracting rational expressions requires common denominators.

Example: $\frac{6}{4a} - \frac{2}{3a}$ (Common denominator = 12a)

Multiply first fraction by $\frac{3}{3}$ and the second fraction by $\frac{4}{4}$ and the new fractions look like:

$\frac{18}{12a} - \frac{8}{12a}$ and then simplify by subtracting the numerators $\frac{18}{12a} - \frac{8}{12a} = \frac{10}{12a}$

Reduce the answer: $\frac{10}{12a} = \frac{5}{6a}$

Simplify each rational expression.

52) $\frac{5x^2-20}{3x^2+5x-2}$

53) $\frac{5x^3}{4y} \div \frac{16y^2}{25x^3} \cdot \frac{16y}{25x^3}$

52) _____

53) _____

54) $\frac{x-2}{8} - \frac{2x+1}{12}$

55) $\frac{2}{x-2} - \frac{1}{x^2+x-6}$

54) _____

55) _____

56) $\frac{x}{x+y} + \frac{x}{x-y}$

56) _____

Simplifying Radicals & Radical Operations

Simplifying Non-Perfect Square Roots

Break down the number underneath the radical into its prime factors. (Make factor tree) Factors that show up twice can be removed from under the radical, and rewritten outside the radical once. Other factors stay under the radical.

Re-multiply any factors outside or under the radical when needed.

Example: $\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5} = 2 \cdot 3\sqrt{5} = 6\sqrt{5}$

Multiplying Radicals

Multiply numbers underneath the radical, and multiply any numbers on the outside of the radical together. Simplify the resulting radical.

Example 1: $-4\sqrt{14} \cdot 2\sqrt{8} = -4 \cdot 2\sqrt{14 \cdot 8} = -8\sqrt{112} = -8\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 7} = -8 \cdot 2 \cdot 2\sqrt{7} = -32\sqrt{7}$

Example 2 **(FOIL!):** $(4 + \sqrt{3})(2 - \sqrt{3}) = (4)(2) - 4\sqrt{3} + 2\sqrt{3} - \sqrt{3 \cdot 3}$

$= 8 - 4\sqrt{3} + 2\sqrt{3} - \sqrt{9}$

$= 8 - 2\sqrt{3} - 3$

$= 5 - 2\sqrt{3}$

Dividing Radicals

Simplify (divide/reduce) the radicands, if possible. Simplify the resulting radical, along with any coefficients.

Example: $\frac{3\sqrt{5}}{\sqrt{45}} \div \frac{\sqrt{5}}{\sqrt{5}} = \frac{3}{\sqrt{9}} = \frac{3}{3} = 1$

Adding and Subtracting Radicals

Numbers underneath the radical must be the same in order to add or subtract terms. This may require you to simplify the radicals first, then add or subtract the coefficients, and keep the common radical.

Example: $-2\sqrt{20} - 2\sqrt{5} = -2\sqrt{2 \cdot 2 \cdot 5} - 2\sqrt{5} = -2 \cdot 2\sqrt{5} - 2\sqrt{5} = -4\sqrt{5} - 2\sqrt{5} = -6\sqrt{5}$

Simplify each radical expression.

57) $\sqrt{224}$

58) $\sqrt{324}$

57) _____

58) _____

59) $\sqrt{3} \cdot 4\sqrt{3}$

60) $(3\sqrt{5} + \sqrt{3})(3\sqrt{5} - \sqrt{3})$

59) _____

60) _____

61) $-10\sqrt{18} - 5\sqrt{32}$

62) $\frac{11\sqrt{6}}{\sqrt{98}}$

61) _____

62) _____